Basis Change Enables Better Fracture Pressure Prediction Via Only Seismic Data

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Abstract

This paper presents the method of predicting fracture gradients using tri-axial compaction, long ignored in the sciences as well as in the petroleum and energy industry in general. The method is entirely theoretical as opposed to the empirical methods of trend lines and pseudo-properties that the industry has been relegated to use for decades. The previous use of empirical techniques in areas where they have not yet been calibrated had appreciable uncertainty. The "Tri-Lateral Compaction Method" is presented here as a method of accurately predicting a basin stress model based on only pre-drill seismic attributes. This method is evaluated by using data from explored basins, checking actual conditions against the method's predictions. It is shown that the new method provides never before seen accuracy in the relaxed basin environments the model was tested in.

Introduction

The bi-lateral constraint equation or other elastic uniaxial strain models have been overwhelmingly in use to predict the minimum stress as a function of depth and rock lithology for projects requiring an underground stress model. Its popularity certainly comes from its simplicity because the parameters needed are easily had from wireline measurements. The models have shown significant accuracy at shallow depths and significant inaccuracies at depth according to Zoback (2007). To compensate pseudo-properties, based on empirical data from offset wells in the basin, were incorporated into most models to produce trend lines according to Bourgoyne et. al. (1986). This model has been reproduced in numerous varieties over many years and yet has left all practitioners desirous of a scientific theory that models observed behavior using properties from pre-drill seismic evaluations and therefore allowing exploratory drilling with precise lower limits on fracture gradients.

The bi-lateral constraint stress model, most commonly used, is based on linear elasticity and assumes that rock is a semi-infinite isotropic poroelastic medium subjected to gravitational loading and zero horizontal strain. The term "bilateral constraint", meaning "zero strain" is akin to assuming the earth is flat. This new model, tri-lateral compaction, acknowledges the earth is round, and compaction occurs exactly as it does; tri-laterally. Sediment is buried and always being forced down into a smaller space that is easily modeled as a square decreasing in area as a function of burial depth. The details are simple as well as globally correct. The model assumes that true elastic behavior begins once the grain matrix is in contact and begins deforming. The model proposes that this begins once pores restrict hydrostatic equilibrium upon this tri-lateral compaction during burial. This state is evident in the stress record obtained from pre-drill seismic attributes, in the form of a reversal of the initial vertical stress loading line. We simply notice the earliest burial stress trend and the first sign of deviation. The burial stress is calculated the normal way by subtracting the pore pressure estimate from the overburden using the Terzaghi (1943) relation:

$$\sigma_v = S_v - P_p \qquad (1)$$

The poroelastic component acts on the spherical pore matrix space and thus the elastic coefficient is expected to be three dimensional (3D) and neither a linear elastic nor planar elastic strain ratio exponent. Hooke's Law (1678) employs a constant k of the first order as expected in the linear realm and yet in the spherical realm where the in-situ stresses express as 3D and the ratio of strain in the poroelastic ratio according to Biot (1941) is in the form:

$$\alpha = 1 - \frac{K}{K_S} \quad (2)$$

Since the K/K_s, bulk modulus term, has no theoretical equation of estimation via seismically attained attributes, at the present time, the author proposes a poroelastic component modeled after the volume of a sphere:

$$Vsphere = \frac{4 \cdot \pi}{3} r^3 \quad (3)$$

Consider that in Biot's Theory of poroelasticity, there is a change in pore volume caused by equal increments of both the pore and confining pressures. This statement takes the form of the proportional relation:

$$\Delta V_{\Delta sphere} \ varies \ as \ \varepsilon^3 \cdot P_p \cdot \sigma_v$$

And

$$\Delta P_{\Delta sphere} \ varies \ as \ \Delta V_{sphere}$$

And therefore

$$\Delta P_{\Delta sphere} \ varies \ as \ \varepsilon^3 \cdot P_p \cdot \sigma_v \quad (4)$$

Since the shrinking and expanding of a pore space is a function of its radius cubed and it is widely accepted that elastic parameters are a function of the stress state, the author constructed a poroelastic term replacing the

$$\frac{K}{K_S} \quad (5)$$

term of Biot's equation (2), with one of a strain ratio cubed as a function of stress. The new poroelastic coefficient replaces the first order bulk modulus ratio with a third order term relating strain cubed to linear overburden stress. Analogous to the 3D "hoop stress" of a cylinder, a pore spherical shell participates in the stress matrix with fluid pressure and rock according to these principles.

Since the use of the Biot Coefficient is for reductions in the spherical stress that we know of as pore pressure (P_p) via the poroelastic effect and commonly utilized as:

$$Pp = \alpha \cdot P_p \quad (6)$$

The reduction takes the form:

$$Pp = P_p - \Delta P_{\Delta sphere} \quad (7)$$

from eq. 4. It follows that:

$$Pp = P_p - (\varepsilon^3 \cdot P_p \cdot \sigma_v) \quad (8)$$

And:

$$Pp = P_p \cdot (1 - \varepsilon^3 \cdot \sigma_v) \quad (9)$$

Now using isotropic Poisson's Ratio as our spherical strain term:

$$\varepsilon = \nu$$

and substituting into eq. 9 we get:

$$Pp = P_p \cdot (1 - \nu^3 \cdot \sigma_v) \quad (10)$$

Replacing the first order bulk modulus ratio with a third order term relating strain cubed to linear overburden stress we obtain the new coefficient:

$$\alpha = 1 - \nu^3 \cdot \sigma_v \quad (11)$$

This is the new formula for poroelastic coefficient containing a dimensionally consistent and readily available strain modulus.

With this derivation in hand we now derive the equation for tri-lateral compaction by combining Terzaghi's Principle, Hooke's Law and Biot's Theory of Poroelasticity. First via Hooke's Law we have:

$$\varepsilon_h = \frac{1}{E}[\sigma_h - \nu \cdot (\sigma_H + \sigma_v)] \quad (12)$$

$$\varepsilon_H = \frac{1}{E}[\sigma_H - \nu \cdot (\sigma_h + \sigma_v)] \quad (13)$$

With these derivations and constitutive equations in hand we now derive the following equation of non-constraint by combining Terzaghi's Principle, Hooke's Law and Biot's Theory of Poroelasticity along with equation 11.

$$S_h = \frac{\nu}{1-\nu}\left(S_v - \propto P_p\right) + \propto P_p + \frac{E\nu}{1-\nu^2}\varepsilon_H + \frac{E}{1-\nu^2}\varepsilon_h \quad (14)$$

The resulting equation assumes there is strain because of the constant decrease in area of a square of rock under multi-lateral burial loads in the stress model (other than tectonic strains).

There still remains the derivation of the non-gravity horizontal strains equation for tri-lateral compaction. The petroleum industry has used the strain terms in equation 14, to add tectonic stress, to empirically match actual values of fracture gradient measured. According to the author these terms are better used with actual and predictable strain in the earth model. As is the concept of this paper these strains are of a square decreasing with burial. Tectonic strains will continue to be accounted for via the same terms added to equation 14 in complete compliance with Hooke's Law, Biot's Theory and Terzaghi's Principle. This will be done in the final form of the equation.

The "tri-lateral compaction" model provides for real and globally correct horizontal strain and is easily understood via the basin model of simple depositional systems. Compaction subsidence models and Magellan's nautical proof of the earth's sphericity lead us to conclude that for sediment deposited at the surface of a basin subsequent burial will cause tri-lateral compaction in the form of decreasing squares:

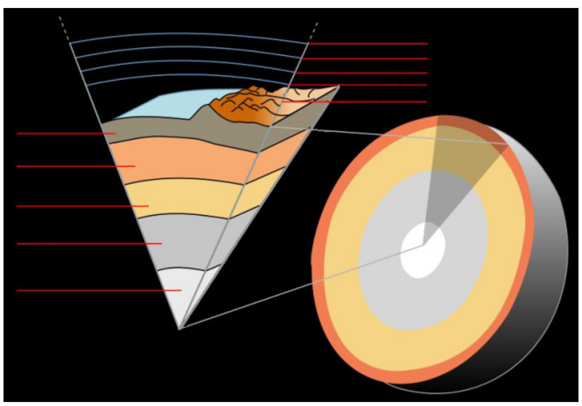

Fig. 1--Graphic depiction of the earth as a sphere and a square plot of terrain decreasing in area with depth to a theoretical point at the core of the earth.

The equations that model this compaction are basically strain derived from the circumference of the earth:

$$Circumference_{\oplus} = C_{\oplus} = 2 \cdot \pi \cdot R_{\oplus} \quad (15)$$

The radius of the earth will be assumed to be isotropic yet as you can imagine you may attempt to get the most accurate estimate for any drilling location in the world since earth isn't perfectly spherical. The horizontal strains are remarkably easy to derive by noting that at the depth of burial the circumference is:

$$Circumference_{depth_{interest}} = C_{d_i} = 2 \cdot \pi \cdot (R_{\oplus} - Depth_{interest}) \quad (16)$$

In order to truly model tri-lateral compaction the depth at which the sand grains actually make meaningful contact and deformation (strain) begins must be defined as the depth at which pores resist escaping pore fluids upon burial to the point that pore pressure prevents further increases in the effective stress of the rock. This is also known as the maximum effective stress, and this point is easily distinguished and captured from the seismic velocities. In the seismic velocity record this depth is notably the maximum velocity. It can also be found by plotting effective stress versus depth. This is easily done because most pore pressure and fracture gradient (PPFG) analysis provides this data already. It is simply a matter of making another column of data using equation 1, and picking the depth where σ_v is a maximum. We will call this depth $D_{\sigma_{v_{max}}}$ or the depth of maximum effective stress, and we will abbreviate the depth of interest as, D_i, and we will consider stress producing strain to begin at this depth. The depth is an obvious choice since poroelastic effects are absent above this depth as witnessed by the straight line trend of effective stress in compacting rock to this depth. Also, $D_{\sigma_{v_{max}}}$ is below surface dominated effects of depositional processes not yet tri-laterally compactional. The circumference at this depth will be designated as:

$$C_{d_i} = \left(C_{\oplus} - C_{D_{\sigma_{max}}}\right) = 2 \cdot \pi \cdot \left(D_i - D_{\sigma_{v_{max}}}\right) \quad (17)$$

$$C_{D_{\sigma_{v_{max}}}} = 2 \cdot \pi \cdot \left(R_{\oplus} - D_{\sigma_{v_{max}}}\right) \quad (18)$$

With this in mind the horizontal strains equation becomes:

$$\varepsilon_{h,H} = \varepsilon_h = \varepsilon_H = \frac{(D_i - D_{\sigma_{v_{max}}}}{(R_{\oplus} - D_{\sigma_{v_{max}}})} \quad (19)$$

This strain equation for tri-lateral compaction assumes, correctly, that a square at the surface shrinks uniformly to a smaller square at the depth of interest. Of course, this assumes isotropy, and yet anisotrophy is accounted for easily and accurately in the tectonic terms. There is no strain above $D_{\sigma_{v_{max}}}$ since dewatering is predominant. This is considered valid, not because there is no rock stress, to the contrary, yet it is increasing to the maximum stress that causes no actual elastic deformation of the grain matrix beyond simply draining the rock matrix of pore fluid. Upon inspection of values common in drilling projects worldwide, the magnitude of order common to these horizontal strain values are $(1-10)\cdot 10^{-4}$ and of course the units are dimensionless as one would expect. These values become meaningful as used with Young's modulus (E), basically the spring constant of Hooke's Law. E values common to the petroleum industry, are on the order of $(1-3)\cdot 10^{6}$ psi and therefore the product of the ε and E are on the order of 100s to 1000s and with units of psi are significant at any depth of drilling wellbores in the energy industry.

The long adhered to paradigm of calculating horizontal stresses on a constant sized cube experiencing increasing overburden pressure alone is absolutely inaccurate and the error magnitudes are a function of depth. While compacting dirt into a funnel is an appropriate analogy of this Tri-Lateral Compaction theory, the past bi-lateral constraint modeling of in-situ stress is similarly analogized as pounding dirt into a coffee can. The vertical stress is translated to horizontal via Poisson's Ratio alone. The global reality is that there is vertical strain and bi-lateral strain from constantly decreasing lateral area.

The Tri-Lateral Compaction Equation

The complete stress model equation of Tri-Lateral Compaction of fluid-saturated porous rock of semi-infinite isotropic poroelastic medium subjected to depositional grain settling and gravitational loading is equation 20:

Eaton (1969) proposed, and empirically derived, Pseudo-Poisson's ratios from offset data to compensate for observed inaccuracies at greater depths. The "tri-lateral compaction" equation 20 requires neither empirical correction nor trend line. The use of uniaxial strain alone, by bi-lateral constraint equations, renders errors as a function of depth as a result of ignoring the reality of tri-lateral compaction.

Target sands at depth tend to be highly dipping and therefore a cos(dip°)·cos(strike°) term must be multiplied times the ε_H and ε_h terms. This compensates for the simple model of strain that assumes formations have 0° dip; essentially flat. The dip° and strike° terms in equation 20 are measured from the direction of the minimum and maximum horizontal orientation respectively as opposed to a reference direction, for simplicity of explanation in this paper.

The precision of the results of the tri-lateral compaction model is remarkable and even a computer aided trendline equation of the log derived Young's Modulus scatter gave Fracture Pressure estimates with deviations of less than 0.95%. S_h estimations using hand-picked values for Young's modulus obtained from a wireline log on a well in the basin a mile away yielded error of less than 0.4%. Of course the scatter is viewed as formation heterogeneity effects that were mitigated by hand picking Young's modulus values from the scatter that would be more appropriate for shales, which the method models. Using seismic of the actual wellbore trajectory will yield the highest precision.

$$S_h = \frac{\nu}{1-\nu}\left(S_v - \propto P_p\right) + \propto P_p + \frac{E\nu}{1-\nu^2}\cdot\frac{(D_i - D_{\sigma_{v_{max}}})}{(R_{\oplus} - D_{\sigma_{v_{max}}})}\cdot\cos(\text{dip}°)\cdot\cos(\text{strike}°) + \frac{E}{1-\nu^2}\cdot\frac{(D_i - D_{\sigma_{v_{max}}})}{(R_{\oplus} - D_{\sigma_{v_{max}}})}\cdot\cos(\text{dip}°)\cdot\cos(\text{strike}°) + \left(\frac{E\nu}{1-\nu^2}\varepsilon_H + \frac{E}{1-\nu^2}\varepsilon_h\right)_{tectonic} \quad (20)$$

Equation 20 is the complete equation for the tri-lateral compaction model and can be equally derived for anisotropy and tectonics. The tectonic term is used to model complex effects such as proximity to structures or salt, inelastic creep, load history, anisotropy, thermal stresses, chemistry, mineralogy, topography, et. al.

Ignoring the poroelastic effects, and assuming there are no horizontal strain nor tectonic stress additions, the above equation reduces to the bi-lateral constraint equation made famous by Eaton (1969):

$$S_h = \frac{\nu}{1-\nu}\left(S_v - P_p\right) + P_p \quad (21)$$

The limitations of equation (21) were that it was found to be accurate at shallow depths and yet underestimating at great depths, and because of its reliance on offset data, it was not appropriate for exploratory drilling projects. Also, complex trajectories and extreme geologic structures with sand buoyancy issues and tectonics, require precision and accuracy in order to design casing setting depths as discussed in depth by Davis (2011).

The new tri-lateral compaction method has been tested against actual well data in the area and shown to deviate very little from a prediction model based on actual data and trend lines drawn from actual data. The fracture strength prediction for the sands is undertaken in a separate buoyancy analysis as is common in a basin model. The tiny deviations from the actual values in the area might be in fact due to the testing procedures and other factors involved in the operations on the drilling rig. Estimations of minimum horizontal stress are well documented, in the literature, as not always being precise, definitive nor conclusive as a result.

Seismic Indications of Strain and Faulting

It is acknowledged that an upper limit of *in situ* stress and strain will be related to critical stress and the frictional strength of faults. It is proposed that seismic indications of faulting be used to calibrate the strain relief and be used along with the tectonic term of equation 20 to compensate accordingly. A seismic section presented to scale can give dip of beds and fault angle by simple use of a protractor. These angles, along with other parameters, can be used to constrain critical stress estimations that in turn confine the in-situ stresses of the model. An appropriate equation in factoring the stress and strain tectonic adjustment to the model may be based on frictional equilibrium and be of the form presented by Zoback (2007):

$$\frac{\sigma_1}{\sigma_3} = \left[(1+\mu^2)^{\frac{1}{2}} + \mu\right]^{-2}$$

Appropriate substitutions and uses are left to the reader and perhaps separate treatments of this subject in subsequent literature.

Conclusions

This paper presents the method of predicting fracture pressure of formations using tri-axial compaction. Long ignored in the sciences and petroleum industry, is the reality of the constantly decreasing area of deposited and buried sediment on earth. The new method is entirely theoretical as opposed to the empirical methods of trend lines and pseudo-properties that the industry has been relegated to use for decades. The "Tri-Lateral Compaction Method" as presented here is a method of accurately predicting a basin stress model based on only pre-drill seismic attributes. This method has been evaluated by using data from an explored basin, checking actual conditions against the method's predictions. It is shown that the new method provides accuracy in the relaxed basin environments the model was tested in. Detailing the dip angle and strike, elastic moduli, and sand pore pressures provides precision. The fracture gradients predicted are in fact S_h and provide one if not both of the most difficult determinations for an adequate geomechanical model in relaxed basins. This method will continue to be tested and extrapolated for use in all stress regimes and basins of the world. Refinements and additions are expected and will be eagerly anticipated for their universal utility and theoretical basis free from the constraints of offset calibration of empirically determined trends. Of course basin modeling of this type is based on relatively impermeable media of shale and the calibration of sand pressures vital to the sand fracture strengths is an integral part of a basin model. The accuracy of the calculations of the sand fracture pressures are highly dependent on the accuracy of the buoyancy calculations which in turn are dependent on an accurate estimation of the fluid columns in the sands. As long as the exact content of the sands in the basin remain speculative the prospect of drilling virgin sands retains the elements of risk, danger, hope and uncertainty. The fluid contents of unexplored basins remain difficult to guarantee with certainty even using our state of the art equipment, methods and reasoning, and yet those unknowns comprise the exact content of our dreams and aspirations as well as the pores of the targets we explore. The diligence and painstaking has its rewards. It is acknowledged that an upper limit of horizontal stress and strain will be related to critical stress and faulting. It is proposed that seismic indications of faulting be used to calibrate the strain relief and be used along with the tectonic term of equation 20 to compensate.

Nomenclature

σ_v = vertical effective stress
σ_h = minimum horizontal effective stress
σ_H = maximum horizontal effective stress
σ_1 = generalized maximum (principle) effective $in-situ$ stress
σ_2 = generalized intermediate effective $in-situ$ stress
σ_3 = generalized minimum effective $in-situ$ stress

S_v = overburden stress
S_h = minimum horizontal stress
S_H = maximum horizontal stress
P_p = pore pressure (αP_p)
P_p = pore pressure independent of poroelasticity
α = poroelastic coefficient
$\frac{K}{K_s}$ = bulk modulus ratio used in the traditional equation of Biot's coefficient (1 - $\frac{K}{K_s}$)
$\Delta P_{\Delta sphere}$ = change in pressure vs. volume of sphere and effective stress of rock matrix
$Vsphere$ = volume of a sphere
ε = strain as defined in classical theoretical mechanics solids
v = Poisson's Ratio = vertical strain/lateral strain
E= Elastic Modulus=Young's Modulus=axial stress/resultant strain
ε_h = strain in the direction of the minimum horizontal stress
ε_H = strain in the direction of the maximum horizontal stress
σ_h = minimum horizontal stress
σ_H = maximum horizontal stress
$R_\oplus$ = radius of the earth ~ 20,903,520 ft
$C_\oplus$ = $Circumference_\oplus$ of the surface of the earth = 2·π·$R_\oplus$ ~ 131,340,690 ft
C_{d_i} = circumference at some depth below the surface of the earth
$C_{D_{\sigma max}}$ = circumference at depth at which the vertical stress is maximum for the total plan interval.
$D_{\sigma_{v max}}$ = depth at which the vertical stress is maximum for the total interval planned to be drilled.
D_i = depth of interest
$\mu = \text{coefficient of friction of linearized Mohr} - \text{Coulomb criterion.}$

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