Understanding Casing Design Loads and Making Sense of the Nomenclature

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Abstract
In this paper we explain the differences between biaxial and triaxial and the API method in regards to casing design. In order to do this we had to explain nomenclature common in our industry and clear up confusion associated with terms used in both cases as a result. First we had to distinguish between casing design and load resistance and make clear that triaxial, biaxial and the API method refer to load resistance criterion. Second we had to distinguish between collapse, burst and axial load resistance. Third we had to show that axial is simply yield strength of the pipe. Fourth, we showed how burst has choice of API method (Barlow's) and that it isn't either triaxial nor biaxial yet sort of uniaxial and the Von Mises, or triaxial, criterion. Fifth, we did inform that collapse has four failure modes and sixth, for yield this is derived from triaxial criterion and yet when axial is set to zero this isn't uniaxial, it is triaxial with one of the three axis set to zero and so possibly biaxial is applicable and with axial load non zero this isn't biaxial it is triaxial. This is contrary to common opinion that collapse with axial load considered is biaxial and collapse without axial is uniaxial. Lastly, we showed the times when Barlow's is acceptable and the times in which it deviates significantly from the more accurate and cumbersome Von Mises Criterion.

Introduction
Let's first make one thing perfectly clear and then move on to clearing up a distinction about "Biaxial" and its use to describe the "API Published Rating" (API method) and the fact that it is one in the same as Barlow's equation. First all of this discussion about uniaxial, biaxial, triaxial, API method, and Barlow's equation is concerned with material strength of casing in regards to load resistance and in a sense really is only one half of casing design. The reason to mention this is that this helps to clarify this in advance because it can get confusing if not done in advance. Of course casing design involves many aspects including selecting casing setting depth, then hydrostatic and other geomechanical loads, working loads during casing emplacement and then the internal, lifecycle of the well, loads. Once these loads have been modelled the strength of the casing must be matched to these design loads and that is what we are discussing here today: load resistance of casing and the different methods to arrive at these strengths to use in our overall casing design.

These concepts are all in regards to the material strength in casing design, heretofore referred to as "load resistance" and not the calculation and modelling of the loads that this material will "resist" except as needed to apply these "loads" to determine the material strength of the casing according to the two most often used criterion. This paper won't delve into the hydrostatics and nuances of casing design loads and yet only explain the different methods of determining the load resistance of the casing used to withstand the loads and how and why these methods vary and when this variance is the greatest and why this is so. Knowledge of this, it is hoped, will be most useful and ensuring that the most exact determination of load resistance of the casing in every casing design.

Loads and Combined Loads and Making Sense of Common Nomenclature In Casing Load Resistance Calculations.

Combined load was indeed in reference to how “Bi-axial” load has been used in the past by others and yet "Tri-axial" loading is the penultimate combined load and is in reference to Von Mises use of the Lame Equation to predict casing strength under every concievable load simultaneously or separately. Would "Biaxial" better describe using Von Mises Criterion and assuming one axis is zero? Perhaps, and if so then using "Biaxial" to describe Barlow's equation and results will be different from using Von Mises Criterion with the appropriate thirds axis zero as will be described in detail below.

There has been much confusion as to the terms “Biaxial”, “API Method”, in regards to Barlow’s equation used by the API and published and utilized by many in the petroleum industry. The way Barlow’s equation uses “combined load” is by adding the effect of axial load to burst/collapse criterion and it is done like this:

In the much used SPE Textbook Series, Vol. 2, "Applied Drilling Engineering" (pp 308-311), the "combined load" is presented by Holmquist and Nadia, "A Theoretical and Experimental Approach to the Problem of Collapse of Deep-Well Casing," Drill. and Prod. Prac., API, Dallas (1939) 392. I've not read this paper and yet will, yet upon a superficial observation this derivation seems to be purely the Von Mises "Triaxial" Criterion. With this in mind then, this is "triaxial" and not "biaxial" because all three axis, radial, hoop, and axial are used to derive the equation used to model the effects of axial load on burst and collapse and vice versa the radial pressure (burst and collapse) effects on axial load resistance.

There is even more confusion over the use of “Biaxial” as it is misused to point to the use of Barlow equations in a uniaxial material load resistance calculation. There is uniaxial, biaxial and triaxial. Simon, again, had a very coherent answer to this worth repeating, “In my experience 'biaxial' is most often associated with collapse-load design, and taken to mean the reduction of pipe collapse resistance as tension increases; it is under collapse loads that the pipe design limit will often also be less than a theoretical yield limit.” Of course “uniaxially” collapse loading is simply the external pressure minus the internal pressure or the one axis of triaxial design referred to as “radial stress”. Of course tension adds to burst resistance and detracts from collapse resistance of pipe. These are two axis, radial and axial stress and therefore is the term “Biaxial” is often used and yet as discussed above the term is confusing and scientifically wrong and the API method in regards to collapse (the load resistance we are discussing since it detracts whereas the burst load resistance is enhance with tension) utilizes Von Mises Triaxial Criterion for the "yield-strength" mode of failure and empirical relations for the other three modes of failure; plastic, transition, and elastic, with internal pressure set to zero. In a sense the "yield strength" mode of failure for collapse might be considered to be "biaxial" since one of the three Von Mises axis is zero and yet don't be confused this is different from the API Burst criterion which uses Barlow's and is in no way triaxial, or biaxial and yet is a uniaxial model. The most common starting point of casing design in the past has thus been “uniaxial”. The use of terms in the past has been a source of confusion we can get past with a clear review of the facts.

Derivation of the Barlow formula and why API RP 7G calls this “Biaxial Design”

The Barlow formula is a simplification of Lame's formula for thin walled pipes/ cylinders. The book by AP Moser (Buried Pipe Design) gives the derivation and arrives at S = PD/2t where D is in fact the average diameter. This makes sense if you consider that there is a stress profile across the pipe wall with highest stress at the inside edge. So the average stress occurs in the material at the centre (ie. Do - t). The above book has a couple of good sketches illustrating this.

Outer diameter is however quoted in the Barlow equation in some contexts - for example in ASME 31.8. This is apparently a simplification (infact nominal outside diameter is used which is actually a bit smaller than the real outer diameter). If using the formula to calculate wall thickness the answer will be slightly conservative but then you will generally look up a standard wall thickness (eg. from the table in ANSI B36.10) so you will likely arrive at the same answer anyway.

9.3.1.2 - The Barlow formula for internal fluid pressure gives results on the side of safety for all practical thickness ratios. This formula is similar to the 'common formula' (which uses the mean diameter instead of the outside diameter). In the common formula, it is assumed that the hoop stress is uniformly distributed across the cylinder wall. This condition does not hold, except in the case of pipes having walls of infinitesimal thickness.

While Barlow's formula is widely used because of its convenience of solution, it was not generally considered to have any theoretical justification until formulae based on the maximum energy of distortion theory showed that for thin walled pipe with no axial tension, Barlow's formula is actually theoretically correct. (See Lester, C. B., "Hydraulics for Pipelines", Oldom Publishing Co., Bayonne, New Jersey, 1958, p 61.)

Barlow's equation is what you are refering to as "Biaxial" and it isn't "Biaxial" in the sense of using two of three Von Mises axis. This confusion began with SPE 14727 that used "Barlow" and "Biaxial" as interchangeable. API TR 5C3 6.6.2.1 states: "The Barlow Equation for pipe yield, which is the historical API equation, is based on a one-dimensional (not triaxial) approximate equation of the von Mises yield condition, combined with an approximate expression for the hoop stress in the pipe. In essence, the Barlow Equation approximates the hoop stress and then equates this approximation to the yield strength. This approximation is less accurate than the Lamé Equation of yield used in Von Mises Equivalent (VME) Stress (Triaxial Stress). Because the Barlow Equation neglects axial stress, there is no distinction between pipe with capped ends, pipe with open ends or pipe with tension end load." Those are not my words, because my opinion of the scientific basis of the Barlow equation is that it works perfectly only if there is no casing since it deteriorates in accuracy with wall thickness which implies that the more casing there is the less accurate the prediction of strength is with maximum accuracy at precisely no casing wall thickness at all. Zero; nada. No casing.

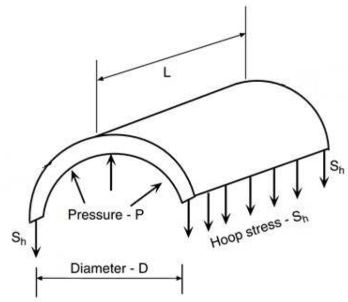
My opinion is that triaxial loading (Von Mises Ellipse of Plasticity) should be used all of the time yet keep in mind there are times when actual casing material limits (from testing) will fall within the VME ellipse of plasticity and in those cases one might take special care to design to the material limits of the casing and not the VME ellipse, although the triaxial theory is arguably more accurate than the API burst formula, so the casing should not fail in this area of the ellipse between the VME limit and the casing material limit. However, it may still mean operating at a higher pressure than the casing has been tested at in the mill. Another argument for avoiding this area is that burst/collapse disks and completion equipment (for example a side pocket mandrel for production tubing) will not be subjected to the same triaxial loads as the casing, but may have the same burst rating.

Now with that said, let's keep in mind that some don't have access to the computer programs that make Triaxial design possible and so if that was the case I would definitely use combined loads if the design was sufficiently close to the safety factor and anytime there is a detrimental effect not considered with uniaxial loading. These combined load corrections are fairly simple and straightforward and easily done on a spreadsheet.

Okay now when to use triaxial over biaxial according to an actual study, in SPE 14727-MS, it is concluded the Triaxial Design procedure should be used when surface pressures exceed +/-12,000 psi. This is based on comparing a series of designs for surface pressures ranging from 3,300 to 21,000 psi. By first designing these strings using the Biaxial/Barlow approximation and then checking them with the Triaxial/Lame' procedure, it is noted that the Biaxial/Barlow design is in considerable error above +/-12,000 psi and may be unsafe above +/-15,000 psi.

There is even more confusion over the use of “Biaxial” as it is misused to point to the use of Barlow equations in a uniaxial material load resistance calculation for burst resistance and also to describe the use of the triaxial Von Mises Criterion with axial load and internal pressure set at zero for yield-strength failure mode by the API in API TR 5C3 for collapse resistance of casing. There is uniaxial, and triaxial and yet "biaxial" is not actually proper use of a term with scientific meaning in a colloquial sense to describe two main directions of force when in fact there are three axis involved in combined stress equations in use: radial, hoop and axial; and thus this is "triaxial" design. Simon had a very coherent answer to this worth repeating, “In my experience 'biaxial' is most often associated with collapse-load design, and taken to mean the reduction of pipe collapse resistance as tension increases; it is under collapse loads that the pipe design limit will often also be less than a theoretical yield limit.” That use is indeed often referred to as "Biaxial" and yet the equation used by the API for collapse is indeed the Von Mises Triaxial Criterion for "yield-strength" collapse and therefore is in no way "Biaxial" and therefore the term is used in error. This is different from the API use of Barlow's equation for burst yield. Of course collapse loading is simply the external pressure minus the internal pressure or the one axis of triaxial design referred to as “radial stress” and then sets the internal pressure and axial load to zero and yet the "hoop stress" axis is comprised of the same internal and external pressures as the "radial stress" term and therefore this distinction is often missed by professionals using the term "biaxial". Of course tension adds to burst resistance and detracts from collapse resistance of pipe and yet although this seems to be "biaxial" it is indeed using the Von Mises Triaxial Criterion in the case of the API yield-strength failure mode. These three axis radial, hoop, and axial consist of terms for radial pressure and axial stress and therefore the term “Biaxial” is often used and yet the hoop stress axis uses the same internal and external pressure terms that the radial stress axis uses and so is derived as triaxial. The API method in regards to collapse (the load resistance we are discussing since it detracts whereas the burst load resistance is enhance with tension) utilizes Von Mises Triaxial Criterion for the "yield-strength" mode of failure and empirical relations for two of the other three modes of failure; plastic, transition, and elastic, with internal pressure set to zero. Plastic and transition failure modes are empirically derived and the minimum elastic collapse pressure equation was derived from the theoretical elastic collapse pressure equation developed by Clinedinst. In a sense the "yield strength" mode of failure for collapse might be considered to be "biaxial" since one of the three Von Mises axis is zero and yet don't be confused this is different from the API Burst criterion which uses Barlow's and is in no way triaxial, or biaxial and yet is a uniaxial model. The use of terms in the past has been a source of confusion we can get past with a clear review of the facts.

Clearly my opinion is that SPE 14727 is in error in proclaiming the Barlow equation or better known as the API method of calculating the strength of casing as Biaxial. It is not using two axis of the Triaxial, radial, hoop and axial stresses and yet it is simply using one dimension of the two radial forces and the materials resistance to this more as a pulling of two thin walls than as a hoop stress. Barlow's derivation uses:



Considering the one-half section of the casing and balancing the forces acting on the two rectangular areas L x t, against the internal pressure on the projected area D x L which is therefore,

$$P \times D \times L = S_h \times L \times t \times 2$$

Solving for S_h, we get the derivation as

$$S_h = \frac{PD}{2t}$$

Whether one wants to argue that this is hoop stress and radial stress it is clearly not considering the balance of Von Mises radial forces and therefore is not “Biaxial”. If it was to consider the balance of radial forces the external pressure would be acting on the outside diameter of the casing and the internal pressure would be acting on the inside diameter of the casing and this is resolved by Barlow by assuming the internal pressure is negative on the outside. Why do I say this? Because the Barlow equation in essence is using the outside diameter and this surface in reality only pertains to external pressure. There is no inside diameter in the equation and the only way this can be is to assume internal pressure is zero. So Barlow’s equation is essential pulling the casing in half using a negative external pressure. The only time this represents physical reality is when the thickness is essential zero meaning when there is no casing (ha ha!). As a result of this Barlow’s equation works best when there is no casing at all and as you might expect as the thickness increases the equation loses validity and this is indeed expressed in API 5C3 by the following:

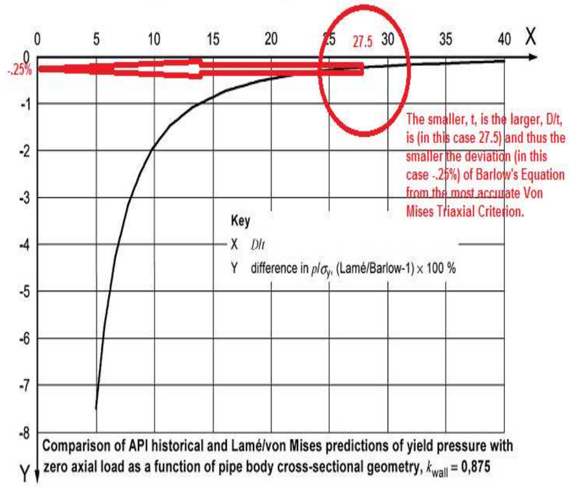
Comparison of historical, one-dimensional yield pressure design equation with openend Lamé Equation for internal pressure with zero axial load

The Barlow and the Lamé Equations are compared by plotting the difference between the Lamé and API historical equations as a percentage of the Barlow Equation, e.g. $[(p/f_y)_{Lamé}/(p/f_y)_{Barlow} - 1] \times 100$ %, for the range of diameter:thickness ratio values typical of oil field tubulars.

Two significant conclusions are:

a) for stated yield stress and cross-sectional dimensions, the Barlow design equation predicts a higher internal pressure resistance than the Lamé Equation for open-ended pipe;

b) the difference between the limit pressures predicted by the two equations is less than 8 % for the range of diameter:thickness ratios typical of oil field tubulars ($D/t > 4,9$).



Comparison of API historical and Lamé/von Mises predictions of yield pressure with zero axial load as a function of pipe body cross-sectional geometry, k_{wall} = 0,875

It is hoped that this clears up the confusing use of "Biaxial" (its not!) to describe Barlow's Equation, and also shows us clearly the API method is most accurate when there is no casing at all and all other times we should use Triaxial Von Mises Criterion for precision.

triaxial design loads becomes critical to consider "when the pipe and/or connection design limit is lower under combined load than the simple uniaxial rating would suggest; conventionally the combination of net internal pressure and axial compression, and net external pressure and tension.", and addressing the confusion over the term "biaxial" to describe casing design.

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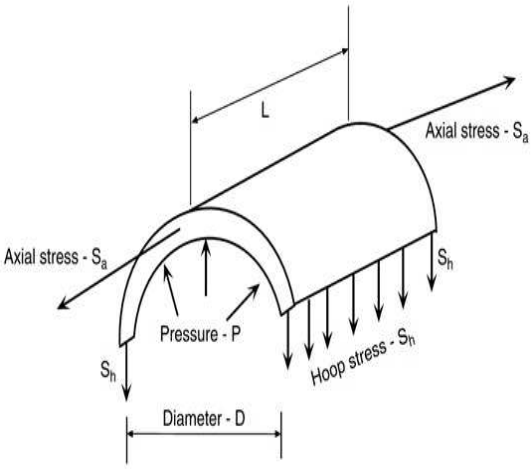

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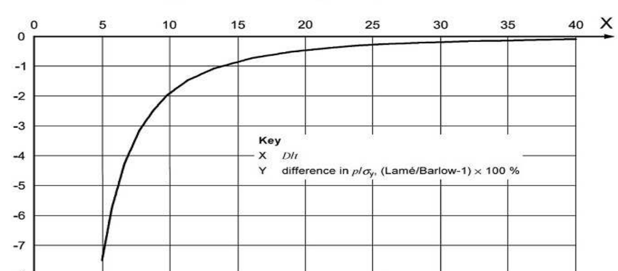

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So when is it most important to use triaxial because of errors creeping in? It was mentioned a high pressures is this right and why? Yes, and because most often high pressures are encountered in the deepest portions of wellbores with the smaller ID casings. Is there another factor? Yes, at higher pressures the wall thickness, t, will be higher because of the need for greater burst resistance and therefore with an increase in, t, the ratio, D/t, decreases and adds error to the Barlow equation. Let's see a comparison of the D/t ratios of the smaller ID casing strings.

Barlow	18722.22	25000	39772.73	17924.53	31250	26639.34	19193.55	16144.93	20701.3
error to VME	1%	3%	7%	1%	4%	3%	2%	1%	2%
Von Mises	18456.64	24370.3	37299.15	17691.41	30029.92	25878.79	18907.45	15974.52	20342.59
Y	125000	125000	125000	125000	125000	125000	125000	125000	125000
D	4.5	5	5.5	6.625	7	7.625	7.75	8.625	9.625
d	3.826	4	3.75	5.675	5.25	6	6.56	7.511	8.031
t	0.337	0.5	0.875	0.475	0.875	0.8125	0.595	0.557	0.797
D/t	13.35312	10	6.285714	13.94737	8	9.384615	13.02521	15.48474	12.07654

These D/t ratios are in the "10" range and therefore indicating an approximate error in the range of "2%" and also of note is that the graph above referenced in API 5C3 matches our calculations in the above table very closely so it is confirmed that our equations are likely the same ones API used. While this is within the range of most safety factors this result does support the general conclusion of SPE 14727 that the use of triaxial Von Mises Criterion becomes more critical at higher pressures. Let's add to that and suggest that it becomes more critical at higher pressures and smaller OD casing of higher weights. Of course the graph used by API above is empirical.

It is hoped that this clears up the confusing use of “Biaxial” (its not!) to describe Barlow’s Equation, and also shows us clearly the API method is most accurate when there is no casing at all and all other times we should use Triaxial Von Mises Criterion for precision.

(Note from Page 308 of the “red book” that eq. 7.3b reduces to eq. 7.2 when the pipe is subjected only to internal pressure.) (Need to resolve how this affects the comments made above)

The Von Mises Criterion, is triaxial. The three principal stress of the Von Mises stress tensor is: axial (tension and compression), radial (internal and external pressures from hydrostatics), and circumferential (hoop stress related to internal and external pressures from hydrostatics).

$$F_e = F_a - p_i r_i^2 \pi + p_o r_o^2 \pi$$

where F_e is the effective axial force, F_a is the actual axial force in the pipe,

p_i is the internal pressure, p_0 is the external pressure, r_i is the inside radius,

$\Delta p = p_0 - p_i$, and r_0 is the outside radius. The von Mises equation is given by:

$$2\sigma_y^2 = (\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2$$

where σ_r is the radial stress, σ_Θ is the hoop stress, and σ_z is the axial stress, given by:

$$\sigma_r = \frac{r_o^2 \Delta p}{r_o^2 - r_i^2} + \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2}$$

$$\sigma_\theta = -\frac{r_o^2 \Delta p}{r_o^2 - r_i^2} + \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2}$$

$$\sigma_z = \frac{F_a}{\pi (r_o^2 - r_i^2)}$$

The von Mises equation now becomes:

$$\sigma_y^2 = \left(\frac{F_e}{\pi (r_o^2 - r_i^2)}\right)^2 + 3\left(\frac{r_o^2 \Delta p}{r_o^2 - r_i^2}\right)^2$$

This is the equation of an ellipse with F_e as the X axis and Δp as the y axis.

API Collapse

The collapse (a radial load of internal minus external) in one axis and the axial load (tension in the string) is the other axis is misleading many to think of this as "biaxial", as discussed above since "hoop stress" is the third axis and uses the same internal minus external pressure term. Considering this as a “biaxial” design would be using the fact that increasing tension decreases the collapse resistance of the pipe or in other words the axial load affects the radial resistance and yet the hoop resistance is involved as well. It is triaxial.

Barlow’s equation is however only used to determine burst by the API. Collapse via yield or “yield-strength” collapse is determined by using triaxial Von Mises Criterion with the internal pressure set to zero and the axial load zero as well (ie. Uniaxial) it can easily be shown that Von Mises Triaxial equation for collapse reduces to (eq 7.11 reduces to eq. 7.4a in “red book” pg 310) the “yield-strength” collapse equation used by the API in API TR 5C3.

Interestingly, torsion and bending also play a part in loading casing and yet are absent from the simplest design cases. Bending is generally acknowledged and brought in as an additional compressive axial load on the concave side of the bend and an additional tensile load on the convex side of the bend. Torsion, on the other hand is rarely considered in casing design yet lets consider it for a moment. With the common practice of rotating liners while cementing it is possible to “trap torque” into a liner and cement it in place with this torsional load. Torsional loads are seen as a “shear stress” and must be added into the Von Mises Criterion as such. An easy entry point would be to convert this “combined load” using equations such as that found in API RP 7G for combined loads of drill pipe thus reducing the tensile strength of casing by adding torsion.

A.8.2 TORSION AND TENSION

$$Q_T = \frac{0.096167J}{D}\sqrt{Y_m^2 - \frac{P^2}{A^2}} \qquad (A.15)$$

where

Q_T = minimum torsional yield strength under tension, ft-lb.,

J = polar moment of inertia

$= \frac{\pi}{32}(D^4 - d^4)$ for tubes

$= 0.098175\,(D^4 - d^4)$,

D = outside diameter, in.,

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Y_m = material minimum yield strength, psi,

P = total load in tension, lbs.,

A = cross section area, sq. in.

This reduction is more significant for smaller tubes than larger ones. For example with 9-7/8" 62.8# Q-125 liner rotated to 60,000 ft-lbs (highly unlikely) of torque and trapped the resultant reduction of tensile strength would be only 1%. With N-80 the torque needed for 1% reduction is only 42,000 ft-lbs. This effect will be larger, of course, for smaller casing such as 5" 18# J55 where 15,000 ft-lbs has a 1% reduction of tensile strength and even more for 3-1/2" 7.7# J-55 where 1,570 ft-lbs of torsion reduces the tensile strength 1% and since the uniaxial torsional strength of that tube is ~11,100 ft-lbs. and the connection is likely to be higher than than and so torquing this tube many times past 1,570 ft-lbs would be operationally probable resulting in a significant reduction in the overall axial strength of this pipe in the casing design if this torque is trapped and so keep this in mind on critical applications where torsional loading is added to pipe especially in the case of small diameter casing and tubing of softer grades. Is Torsion a completely different axis from radial, axial, and circumferential? Is this Quadraxial design? No, it is pure shear stress, and so perhaps we might call it Shear Inclusive Triaxial Casing Design and yet how significant it is in any particular scenario is left up to you to discover, and where you add this torsional shear stress, as a component to the axial stress or elsewhere in the Von Mises equation, is left up to you to decide. Yet consider this:

Load scenario	Restrictions	Simplified von Mises equation
General	No restrictions	$\sigma_v = \sqrt{\frac{1}{2}[(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2 + 6(\sigma_{12}^2+\sigma_{23}^2+\sigma_{31}^2)]}$
Principal stresses	$\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$	$\sigma_v = \sqrt{\frac{1}{2}[(\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2]}$
Pure shear	$\sigma_1 = \sigma_2 = \sigma_3 = 0$ $\sigma_{31} = \sigma_{23} = 0$	$\sigma_v = \sqrt{3}\sigma_{12} = \tau = \frac{Tr}{J}$ J = polar moment of inertia $= \frac{\pi}{32}(D^4 - d^4)$ for tubes $= 0.098175\,(D^4 - d^4)$
Uniaxial	$\sigma_2 = \sigma_3 = 0$ $\sigma_{12} = \sigma_{31} = \sigma_{23} = 0$	$\sigma_v = \sigma_1$

And assuming the torsion is pure shear added to the general equation the new triaxial design with pure shear becomes:

$$\sigma_y = \sqrt{\frac{1}{2}[(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2 + 6(\tau)^2]}$$

With Torque and Bending added this becomes:

$$\sigma_e = [\sigma_r^2 + \sigma_h^2 + (\sigma_a + \sigma_b)^2 - \sigma_r\sigma_h - \sigma_r(\sigma_a + \sigma_b) - \sigma_h(\sigma_a + \sigma_b) + 3\tau_{ha}^2]^{1/2}$$

with

$$\sigma_r = [(p_i d_{wall}^2 - p_o D^2) - (p_i - p_o) d_{wall}^2 D^2 / (4r^2)] / (D^2 - d_{wall}^2)$$

$$\sigma_h = [(p_i d_{wall}^2 - p_o D^2) + (p_i - p_o) d_{wall}^2 D^2 / (4r^2)] / (D^2 - d_{wall}^2)$$

$$\sigma_a = F_a / A_p$$

$$\sigma_b = \pm M_b r / I = \pm Ecr$$

$$\tau_{ha} = Tr / J_p$$

where

A_p is the area of the pipe cross section, $A_p = \pi/4\,(D^2 - d^2)$;

c is the tube curvature, the inverse of the radius of curvature to the centreline of the pipe;

D is the specified pipe outside diameter;

d is the pipe inside diameter, $d = D - 2t$;

d_{wall} is the inside diameter based on $k_{wall}\,t$, $d_{wall} = D - 2k_{wall}\,t$;

E is Young's modulus;

F_a is the axial force;

I is the moment of inertia of the pipe cross section, $I = \pi/64\,(D^4 - d^4)$;

J_p is the polar moment of inertia of the pipe cross section, $J_p = \pi/32\,(D^4 - d^4)$;

k_{wall} is the factor to account for the specified manufacturing tolerance of the pipe wall. For example, for a tolerance of –12,5 %, $k_{wall} = 0{,}875$;

M_b is the bending moment;

p_i is the internal pressure;

p_o is the external pressure;

Summary

This paper addressed the issue of ballooning in drilling and gave a geomechanical explanation and means for understanding, predicting and mitigating the issues. It is my belief that in better understanding the phenomena of ballooning communicating in interdisciplinary teams and conveying issues and instructions between the rig and the office will be greatly enhanced. The result should save time in diagnosing the problems and in implementing the solutions and also result in much safer operations and better plans. In most cases ballooning can be predicted for any trajectory and casing setting depth design using the concept of the FC and an alternative trajectory and/or casing setting depth designed that will avoid the worst conditions or eliminate the ballooning entirely. By changing the trajectory and/or casing setting depths the well designer may place wellbore locations in advantageous positions relative to the PC and FC. If this concept is used to prevent wellbores with extremely high risk of ballooning, these screening and design methods will be responsible for eliminating one of the most, if not the most significant, causes of well control incidents and NPT in the industry.

Nomenclature

$\propto$ = poroelastic coefficient

BT = Ballooning Threshold = $P_{f_{min_{shale}}} - P_{p_{max_{sand}}}$

$D_{\sigma_{v_{max}}}$ = depth at which the vertical stress is maximum for the total interval planned to be drilled.

D_i = depth of interest

ΔP_{swab}= reduction of bottom hole pressure in a wellbore due to swabbing effect of pulling pipe.

E = Young's Modulus aka Elastic Modulus

ε_H = strain in the direction of maximum horizontal stress tectonic & tri-lateral compaction (TLC)

ε_h = strain in the direction of maximum horizontal stress tectonic & tri-lateral compaction (TLC)

EAD = equivalent annular density includes the added weight of circulating mud and cuttings load.

$\mathrm{EAD_{reduce\ ROP}}$ = equivalent annular density while control drilling at reduced ROP to minimize losses.

ECD = equivalent circulating density includes the friction pressure of the circulating mud system.

ESD = equivalent static density of the pressure a drilling fluid column exerts at the hole bottom.
FC = fracture centroid depth
h_f = fracture height
l_f = fracture length
PC = pressure centroid depth
P_f = fracture pressure
$P_{f_{sand}}$ = fracture pressure of a sand
P_p = pore pressure
$P_{f_{min_{sand}}}$ = Minimum fracture pressure of all sands in any open hole interval
$P_{f_{min_{shale}}}$ = Minimum fracture pressure of all shale in any open hole interval
$P_{p_{max_{sand}}}$ = Maximum of all sand pore pressures in any open hole interval
$R_\oplus$ = radius of the earth ~ 20,903,520 ft
S_h = minimum horizontal stress in a normal faulting regime
S_v = overburden stress
TLC = Tri-Lateral Compaction equation
$$S_h = \frac{\nu}{1-\nu}(S_v - \propto P_p) + \propto P_p + \frac{E\nu}{1-\nu^2} \cdot \frac{(D_i - D_{\sigma_{v_{max}}})}{(R_\oplus - D_{\sigma_{v_{max}}})} \cdot \cos(\text{dip}°) \cdot \cos(\text{strike}°) + \frac{E}{1-\nu^2} \cdot \frac{(D_i - D_{\sigma_{v_{max}}})}{(R_\oplus - D_{\sigma_{v_{max}}})} \cdot \cos(\text{dip}°) \cdot \cos(\text{strike}°) + \left(\frac{E\nu}{1-\nu^2}\varepsilon_H + \frac{E}{1-\nu^2}\varepsilon_h\right)_{tectonic}$$
ν = Poisson's Ratio
V_f = fracture volume
w_f = fracture width

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